Prediction of U_{e3} and $\cos 2\theta_{23}$ from discrete symmetry [a]

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We discuss the question why the mixing U_{e3} is small. The natural answer is $U_{e3} = 0$ in some symmetric limit, in which two large mixings are realized. It is possible to force U_{e3} and $\cos 2\theta_{23}$ to be zero by imposing a discrete symmetry. We investigate a special class of symmetries Z_2 and of the consequences of their perturbative violation.

1 Introduction

In the standard model with three families, three mixing angles are free parameters. A lot of studies address the origin of the bi-large mixings of neutrino flavors, which may be a clue to the beyond the standard model, on the other hand, we should also answer the question why the neutrino mixing U_{e3} is so small.

The natural answer is $U_{e3} = 0$ in some symmetric limit, in which two large mixings are realized. There are many examples of symmetries which can force U_{e3} and/or $\cos 2\theta_{23}$ to vanish. Both quantities vanish in the extensively studied bi-maximal mixing *Ansatz* [2,3,4,5], which can be realized through a symmetry [6]. One can also make both U_{e3} and $\cos 2\theta_{23}$ zero while leaving the solar mixing angle arbitrary [7,8]. Alternatively, it is possible to force only U_{e3} to be zero by imposing a discrete Abelian [9] or non-Abelian [10] symmetry; conversely, one can obtain maximal atmospheric mixing but a free U_{e3} in a non-Abelian symmetry or a non-standard CP symmetry [11].

The symmetries mentioned above need not be exact. It is important to consider perturbations of those symmetries from the phenomenological point of view and to study quantitatively [12] the magnitudes of U_{e3} and $\cos 2\theta_{23}$ possibly generated by such perturbations. We discuss a special class of symmetries Z_2 and of the consequences of their perturbative violation. We also

[a] This talk is based on the work collaborated with W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura and H. Sawanaka [1].

study of the specific perturbation which is induced by the electroweak radiative corrections to a Z_2-invariant neutrino mass matrix defined at a high scale. The numerical result of a specific model is presented for this scenario.

2 Vanishing U_{e3} and Z_2 symmetry

Let us construct the neutrino mass matrix in terms of neutrino masses m_1, m_2, m_3 and mixings:

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} , \tag{1}$$

where c_{ij} and s_{ij} denote $\cos\theta_{ij}$ and $\sin\theta_{ij}$, respectively. The neutrino mass matrix M_ν is given in the flavor basis: $M_\nu = U^*_{MNS}\ M_{\mathbf{diagonal}}\ U^\dagger_{MNS}$.

In the standpoint of naturalness, as a dimensionless small parameter s_{13} goes down to zero, the symmetry should be enhanced. In $|U_{e3}| = s_{13} = 0$ limit, M_ν is written as

$$M_\nu = \begin{pmatrix} \tilde{X} & \tilde{A} & \tilde{B} \\ \tilde{A} & \tilde{C} & \tilde{D} \\ \tilde{B} & \tilde{D} & \tilde{E} \end{pmatrix} , \tag{2}$$

where matrix elements are given including Majorana phases ρ and σ:

$$\begin{aligned} &\tilde{X} = c_{12}^2 m_1 e^{-2i\rho} + s_{12}^2 m_2 e^{-2i\sigma} , \quad \tilde{A} = c_{12}s_{12}c_{23}(m_2 e^{-2i\sigma} - m_1 e^{-2i\rho}) , \\ &\tilde{B} = -c_{12}s_{12}s_{23}(m_2 e^{-2i\sigma} - m_1 e^{-2i\rho}) , \quad \tilde{C} = s_{12}^2 c_{23}^2 m_1 e^{-2i\rho} + c_{12}^2 c_{23}^2 m_2 e^{-2i\sigma} + s_{23}^2 m_3 , \\ &\tilde{D} = c_{23}s_{23}(m_3 - m_1 e^{-2i\rho} s_{12}^2 - m_2 e^{-2i\sigma} c_{12}^2), \ \tilde{E} = s_{12}^2 s_{23}^2 m_1 e^{-2i\rho} + c_{12}^2 s_{23}^2 m_2 e^{-2i\sigma} + c_{23}^2 m_3. \end{aligned} \tag{3}$$

There is no explicit symmetry in this mass matrix if there are no relations among matrix elements. However, we obtain the mass matrix with a Z_2 symmetry by putting $\sin\theta_{23} = 1/\sqrt{2}$:

$$M_{\nu f} = \begin{pmatrix} X & A & A \\ A & B & C \\ A & C & B \end{pmatrix} , \tag{4}$$

with

$$\begin{aligned} &X = c_{12}^2 m_1 e^{-2i\rho} + s_{12}^2 m_2 e^{-2i\sigma} , \qquad A = -\frac{1}{\sqrt{2}} c_{12}s_{12}(m_1 e^{-2i\rho} - m_2 e^{-2i\sigma}) , \\ &B = \frac{1}{2}(s_{12}^2 m_1 e^{-2i\rho} + c_{12}^2 m_2 e^{-2i\sigma} + m_3) , \quad C = \frac{1}{2}(s_{12}^2 m_1 e^{-2i\rho} + c_{12}^2 m_2 e^{-2i\sigma} - m_3) , \end{aligned} \tag{5}$$

where

$$S M_{\nu f} S = M_\nu^{(0)} , \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S^2 = 1 .$$

The matrix S is a realization of the discrete group Z_2. It is emphasized that m_1, m_2, m_3, θ_{12}, ρ, σ are arbitrary. In order to respect the symmetry, θ_{23} is maximal, but θ_{12} is not necessarily maximal. The general discussion of this symmetry was given in the previous work [1].

3 Non-zero U_{e3}, $\cos 2\theta_{23}$ from Z_2 breaking

Consider a general perturbation $\delta\mathcal{M}_{\nu f}$ to $M_{\nu f}$ in eq.(4). The matrix $\delta\mathcal{M}_{\nu f}$ is a general complex symmetric matrix, but part of it can be absorbed through a redefinition of the parameters in eq.(4). The remaining part can be written, without loss of generality, as

$$\delta\mathcal{M}_{\nu f} = \begin{pmatrix} 0 & \epsilon_1 & -\epsilon_1 \\ \epsilon_1 & \epsilon_2 & 0 \\ -\epsilon_1 & 0 & -\epsilon_2 \end{pmatrix}. \tag{6}$$

The perturbation is controlled by two parameters, ϵ_1 and ϵ_2, which are complex and model-dependent. We want to study their effects perturbatively, i.e. we want to assume ϵ_1 and ϵ_2 to be small. We define two dimensionless parameters:

$$\epsilon_1 \equiv \epsilon A, \qquad \epsilon_2 \equiv \epsilon' B. \tag{7}$$

Thus, we have the neutrino mass matrix with Z_2 breaking as follows:

$$\mathcal{M}_{\nu f} = \begin{pmatrix} X & A\,(1+\epsilon) & A\,(1-\epsilon) \\ A\,(1+\epsilon) & B\,(1+\epsilon') & C \\ A\,(1-\epsilon) & C & B\,(1-\epsilon') \end{pmatrix}, \tag{8}$$

where we shall assume ϵ and ϵ' to be small, $|\epsilon|, |\epsilon'| \ll 1$.

One finds that, to first order in ϵ and ϵ', the only effect of the $\delta\mathcal{M}_{\nu f}$ is to generate non-zero U_{e3} and $\cos 2\theta_{23}$. The neutrino masses, as well as the solar angle, do not receive any corrections. U_{e3} and $\cos 2\theta_{23}$ are of the same order as ϵ and ϵ'. Define

$$\hat{m}_1 \equiv m_1 e^{-2i\rho}\ , \quad \hat{m}_2 \equiv m_2 e^{-2i\sigma}\ , \quad \overline{\epsilon} \equiv (\hat{m}_1 - \hat{m}_2)\,\epsilon\ , \quad \overline{\epsilon}' \equiv \frac{\hat{m}_1 s_{12}^2 + \hat{m}_2 c_{12}^2 + m_3}{2}\,\epsilon'\ , \tag{9}$$

we get

$$\begin{aligned} U_{e3} &= \frac{s_{12}c_{12}}{m_3^2 - m_2^2}\left(\overline{\epsilon} s_{12}^2 \hat{m}_2^* + \overline{\epsilon}^* s_{12}^2 m_3 - \overline{\epsilon}' \hat{m}_2^* - \overline{\epsilon}'^* m_3\right) \\ &\quad + \frac{s_{12}c_{12}}{m_3^2 - m_1^2}\left(\overline{\epsilon} c_{12}^2 \hat{m}_1^* + \overline{\epsilon}^* c_{12}^2 m_3 + \overline{\epsilon}' \hat{m}_1^* + \overline{\epsilon}'^* m_3\right), \\ \cos 2\theta_{23} &= \mathrm{Re}\left\{\frac{2c_{12}^2}{m_3^2 - m_2^2}\left(\overline{\epsilon} s_{12}^2 - \overline{\epsilon}'\right)(\hat{m}_2 + m_3)^* - \frac{2s_{12}^2}{m_3^2 - m_1^2}\left(\overline{\epsilon} c_{12}^2 + \overline{\epsilon}'\right)(\hat{m}_1 + m_3)^*\right\}. \end{aligned} \tag{10}$$

The induced values of $|U_{e3}|$ and $|\cos 2\theta_{23}|$ are strongly correlated to neutrino mass hierarchies. This makes it possible to draw some general conclusions even if we do not know the magnitudes of ϵ, ϵ'. Remarks are given as follows:

- U_{e3} gets suppressed by a factor $\mathcal{O}(\frac{\Delta_{\rm sun}}{\Delta_{\rm atm}})$ for the inverted or quasi-degenerate spectrum with $\rho = \sigma = 0$. Similar suppression also occurs in the case of the normal neutrino mass hierarchy even when $\rho \neq \sigma$. U_{e3} need not be suppressed in other cases and can be large.

- In contrast to U_{e3}, $\cos 2\theta_{23}$ is almost as large as ϵ, ϵ' if neutrino mass spectrum is normal or inverted. It gets enhanced compared to these parameters if the spectrum is quasi-degenerate.

- In case of the quasi-degenerate spectrum, both $|\cos 2\theta_{23}|$ and $|U_{e3}|$ can become quite large and reach the present experimental limits. The parameters ϵ, ϵ' are constrained to be lower than 10^{-2} for the quasi-degenerate spectrum.

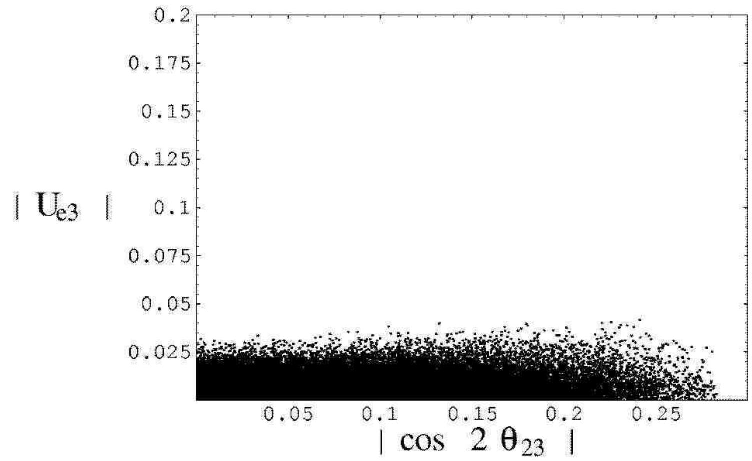

Figure 1: In case of the normal neutrino mass hierarchy with $\rho = 0,\ \sigma = 0$ and $\epsilon, \epsilon' = -0.3 \sim 0.3$.

In our numerical study, the input parameters are randomly varied in the experimentally allowed regions. m_1 was varied up to m_2. On the other hand, ϵ, ϵ' are unknown unless the symmetry breaking is specified, so these are varied in the range $-0.3 \sim 0.3$ with the condition that the output parameters should lie in the 90% CL limit of the experimental data.

In Fig.1, we show the numerical result in the case of the normal neutrino mass hierarchy with $\rho = \sigma = 0$. The $|U_{e3}|$ is forced to be small less than 0.025. The value ~ 0.025 at the upper end arises from the (assumed) bound $|\epsilon|, |\epsilon'| \leq 0.3$. Since $|U_{e3}|$ is proportional to ϵ, ϵ', it increases if the bound on ϵ, ϵ' is loosened. However, $|\epsilon| \leq 0.3$ is a reasonable bound due to assume if Z_2 breaking is perturbative. On the other hand, $|\cos 2\theta_{23}|$ can assume large values as seen from Fig. 1. The present bound $\sin^2 2\theta_{23} > 0.92$ from the atmospheric experiments gets translated to $|\cos 2\theta_{23}| < 0.28$ which constrains $|\epsilon'| \leq 0.2$ in our analyses. The phase dependence is found in the prediction of $|U_{e3}|$, which increases up to 0.075 [1].

We wish to point out an interesting aspect of this analysis. Since U_{e3} is zero in the absence of the perturbation, the CP violating Dirac phase δ relevant for neutrino oscillations is undefined at this stage. CP violation is present through the Majorana phases ρ and σ. Turning on perturbation leads to non-zero U_{e3} and also to a non-zero Dirac phase even if perturbation is real. Moreover, δ generated this way can be large and independent of the strength of perturbation parameters [13].

4 Radiatively generated U_{e3} and $\cos 2\theta_{23}$

The ϵ, ϵ' were treated as independent parameters so far. They can be related in specific models. We now consider one example which is based on the electroweak breaking of the Z_2 symmetry in the MSSM. We assume that neutrino masses are generated at some high scale M_X and the effective neutrino mass operator describing them is Z_2 symmetric with the result that $U_{e3} = \cos 2\theta_{23} = 0$ at M_X. This symmetry is assumed to be broken spontaneously in the Yukawa couplings of the charged leptons. This breaking would radiatively induce non-zero U_{e3} and $\cos 2\theta_{23}$ [14]. This can be calculated by using the renormalization group equations (RGEs) of the effective neutrino mass operator [15,16,17]. These equations depend upon the detailed structure of the model below M_X. We assume here that theory below M_X is the MSSM and use the RGEs derived in this case. Subsequently we will give an example which realizes our assumptions.

Integration of the RGEs allows us [15,16,17] to relate the neutrino mass matrix $\mathcal{M}_{\nu f}(M_X)$ to the corresponding matrix at the low scale which we identify here with the Z mass M_Z:

$$\mathcal{M}_{\nu f}(M_Z) \approx I_g I_t \ (I \ \mathcal{M}_{\nu f}(M_X) \ I \ ) \ , \tag{11}$$

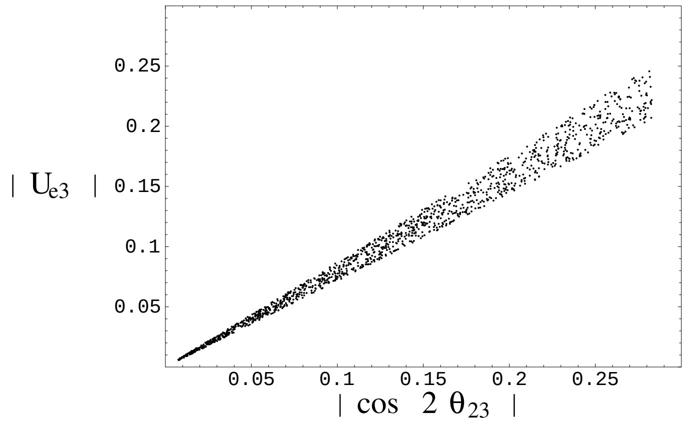

Figure 2: In case of the radiatively broken Z_2 and the quasi-degenerate neutrino masses with $\rho = 0,\ \sigma = \pi/2$.

where $I_{g,t}$ are calculable numbers depending on the gauge and top quark Yukawa couplings. I is a flavor dependent matrix given by

$$I \approx \mathrm{diag}(1+\delta_e, 1+\delta_\mu, 1+\delta_\tau) \quad \text{with} \quad \delta_\alpha \approx c\left(\frac{m_\alpha}{4\pi v}\right)^2 \ln\frac{M_X}{M_Z} \ , \tag{12}$$

where $c = \frac{3}{2}, -\frac{1}{\cos^2\beta}$ in case of the SM and the MSSM, respectively [15]. v refers to the VEV for the SM Higgs doublet.

Since $\mathcal{M}_{\nu f}(M_X)$ is given by eq. (4), we can write $\mathcal{M}_{\nu f}(M_Z)$ as follows when the muon and the electron Yukawa couplings are neglected:

$$\mathcal{M}_{\nu f}(M_Z) \quad = \quad \begin{pmatrix} X & A' & A' \\ A' & B' & C' \\ A' & C' & B' \end{pmatrix} + \begin{pmatrix} 0 & A'\epsilon & -A'\epsilon \\ A'\epsilon & B'\epsilon' & 0 \\ -A'\epsilon & 0 & -B'\epsilon' \end{pmatrix} + O(\delta_\tau^2) \ , \tag{13}$$

where

$$C' = C(1+\delta_\tau) \ \ , \ \ A' = A(1+\frac{\delta_\tau}{2}) \ \ , \ \ B' = B(1+\delta_\tau) \ \ , \ \ \epsilon = \frac{\epsilon'}{2} = -\frac{\delta_\tau}{2} \ . \tag{14}$$

Note that m_1, m_2 and m_3 defined previously are no longer mass eigenvalues because of the changes $A \to A'$, $B \to B'$ and $C \to C'$. Then we get

$$\begin{aligned} U_{e3} &\simeq -\frac{\delta_\tau s_{12} c_{12}}{2(m_3^2 - m_1^2)}\left[m_1^2 + 2m_3\hat{m}_1{}^* + m_3^2\right] + \frac{\delta_\tau s_{12} c_{12}}{2m_3^2 - m_2^2}\left[m_2^2 + 2\hat{m}_2{}^* m_3 + m_3^2\right] \ , \\ \cos 2\theta_{23} &\simeq \frac{\delta_\tau s_{12}^2}{m_3^2 - m_1^2}\left[m_1^2 + 2m_3\hat{m}_1{}^* + m_3^2\right] + \frac{\delta_\tau c_{12}^2}{m_3^2 - m_2^2}\left[m_2^2 + 2\hat{m}_2{}^* m_3 + m_3^2\right] \ . \end{aligned} \tag{15}$$

It is seen that the effect of the radiative corrections is enhanced in the case of the quasi-degenerate neutrino masses with $|\rho - \sigma| = \pi/2$ as previous works presented [16,17]. In the MSSM, the parameter δ_τ is negative and its absolute value can become quite large for large $\tan\beta$. Results of the numerical analysis are shown in Fig. 2 in case of the quasi-degenerate spectrum with $m = 0.3$ eV$, \sigma = \pi/2, \rho = 0$. Both $|U_{e3}|$ and $|\cos 2\theta_{23}|$ can reach their respective experimental bound. We find numerically that $\tan\beta$ is constrained to be lower than 20 in this case. On the other hand, $|U_{e3}|$ reaches at most 0.025 in the normal-hierarchy and inverted-one. The forthcoming experiments will be able to test this relationship between $|U_{e3}|$ and $|\cos 2\theta_{23}|$.

5 Summary

The neutrino mixing matrix contains two small parameters $|U_{e3}|$ and $\cos 2\theta_{23}$ which would influence the outcome of the future neutrino experiments. The vanishing of $|U_{e3}|$ was shown to

follow from a class of Z_2 symmetries of $\mathcal{M}_{\nu f}$. This symmetry can be used to parameterize all models with zero U_{e3}. A specific Z_2 in this class also leads to the maximal atmospheric neutrino mixing angle. We showed that breaking of this can be characterized by two dimensionless parameters ϵ, ϵ' and we studied their effects perturbatively and numerically.

It was found that the magnitudes of $|U_{e3}|$ and $|\cos 2\theta_{23}|$ are strongly dependent upon the neutrino mass hierarchies and CP violating phases. The $|U_{e3}|$ gets strongly suppressed in case of the inverted or quasi-degenerate neutrino spectrum if $\rho = \sigma$ while similar suppression occurs in the case of normal hierarchy independent of this phase choice. The choice $\rho \neq \sigma$ can lead to a larger values ~ 0.1 for $|U_{e3}|$ which could be close to the experimental value in some cases with inverted or quasi-degenerate spectrum. In contrast, the $|\cos 2\theta_{23}|$ could be large, near its present experimental limit in most cases studied. For the normal and inverted mass spectrum, the magnitude of $\cos 2\theta_{23}$ is similar to the magnitudes of the perturbations ϵ, ϵ' while it can get enhanced compared to them if the neutrino spectrum is quasi-degenerate.

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